Statistical characteristics of intermittent liquid film flow

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A free surface liquid film flow was investigated in a range of values of liquid Reynolds number, within which an intermittent regime was observed. Intermittency was characterized by a regular alternation of quiet phases during which the flow displayed non-turbulent behaviour and less quiet phases with turbulent behaviour, randomness and irregularity of the free surface. The invesigated characteristics were: intermittency factor, mean period and lifetime of turbulent bursts, mean and root-mean-square values of film thickness during non-turbulent and turbulent phases. The results were compared to those obtained in a boundary layer and in film flow with intermittency.

Keywords: *intermittency, free surface, burst, turbulence, interfacial transfer*

Introduction

The free surface of a liquid film is the interface in many gas–liquid systems[1–5]. The structure in the vicinity of the free surface has an important role in transfer mechansims: (1) in increasing the interfacial area due to the free surface motions; (2) in increasing the exchange surface renewal rate by strong mixing produced by turbulent bursts. The role of intermittency is very often discussed in momentum, mass and heat transfer studies, but few reports deal with its statistical characteristics in liquid film flow.

Experimental details

Experimental conditions

In this work an aqueous soda solution flowed by gravity from a constant level tank in a rectangular channel 1.10 m long, 0.206 m wide and 0.05 m high. For the aqueous soda solution used, the following variables were evaluated.

(a) The kinematic viscosity ν was measured before every experiment. It varied with temperature in the range 0.009 to 0.010 $cm^2\,s^{-1}$ (or stokes).
(b) The superficial tension σ was also determined and was about 73.6 $dyn\,cm^{-2}$ (0.0736 $N\,m^{-1}$).
(c) The liquid volumetric discharge Q per unit channel width was in the range 0.18 to 92.8 $cm^2\,s^{-1}$.
(d) The liquid Reynolds number $Re_L(=Q/\nu)$ ranged from 18 to 10000.

Measurements were made at various positions x in the channel: 0.20 m, 0.60 m and 0.90 m for inclination angle θ of about 3°.

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Liquid film thickness measurement

Taking into account the free surface and small film thickness, we developed a method of instantaneous thickness measurement without contact with the film, using a capacitive probe (Fig 1). The air gap between a rectangular plane electrode (Fig 1(c)) and the liquid free surface acts as a dielectric of thickness e for a capacitor of capacitance c. The free surface motions produce changes of e, c, and f, the resonance frequency of an oscillatory circuit which includes c. A reactance converter gives a voltage signal $V(t)$ for an instantaneous liquid film thickness $b(t)$.

Frequency analysis of such a signal[6] showed that the film thickness fluctuation frequency was in the range of a few hertz (0 to 10 H_3), for laminar or wavy laminar film, to one hundred hertz for fully developed turbulent film flow.

The acquisition at 200 Hz of 51 200 values of the voltage signal $V(t)$ yields the ensemble averages: mean liquid film thickness $\bar{b}$; root-mean-square values of film thickness fluctuations, b' for the whole flow, b'_{nT} for non-turbulent phases, and b'_T for turbulent phases.

The capactive method used here is made accurate by the choice of a high value of the frequency of the central reactance converter oscillatory circuit: about 5 MHz. The non-linearity effect of the capacitive displacement probe is reduced by a sampling technique, and Lagrange interpolation used to obtain the film thickness $b(t)$ from the signal $V(t)$. Calibration values of oscillatory frequency f versus air-gap thickness e were used in interpolation, leading to instantaneous values $b(t)$ of film thickness. The thickness measurements are made within an accuracy of less than 4 μm.

Intermittency factor and determination of conditional averages

In the intermittency regime, the liquid film thickness fluctuations have amplitudes and frequencies higher in periods of turbulence than in periods of non-turbulence, as shown on the $V(t)$ signal recordings (Fig 1).

The processing of the voltage signal $V(t)$ input to the computer consists of several steps (Fig 2): (1) band-pass filtering (50 to 200 Hz); (2) full wave rectifying; (3) linear detection by low-pass filtering (0 to 40 Hz); (4) logical filtering by threshold analysis. The signal obtained after this last step allows us to determine the time interval T_{B_i} between two successive bursts, the lifetime τ_{B_i} of the ith burst ($i=1, 2, \ldots, N$), the intermittency factors $\gamma_i=\tau_{B_i}/T_{B_i}$, the fraction of time during which the flow is in intermittence, and γ the ensemble average of γ_i.

Evolution of γ with the threshold S allows determination of the value S_1 corresponding to the non-turbulent/turbulent interface, marked by a prompt jump of the slopes of the curves $\gamma(S)$, the probability function of S, and $d\gamma/dS$, the probability density of S, as shown in Fig 3.

Comparing the logical filtered signal to the voltage signal $V(t)$, we obtain conditional averages of film thickness $\bar{b}_{nT}$ for periods of non-turbulence, $\bar{b}_T$ for periods of turbulence, and corresponding rms values of film thickness fluctuations, b'_{nT} and b'_T.

Experimental results and discussion

We report here only the results of measurements at the position $x=0.60$ m. The graph of liquid film thickness fluctuations rms value b' versus liquid Reynolds number Re_L (Fig 4) shows the ranges of various flow regimes:

- smooth laminar or wavy when $18<Re_L<200$, Range (1);
- laminar to turbulent transition for $200<Re_L<1000$, Range (2);
- turbulent for $Re_L>1000$, Ranges (3) and (4).

Intermittency is observed in Range (3) where Re_L changes from about 1800 to a value, Re_1, of 3500 to 5000 for measurements near the channel exit or entry, respectively. We have $Re_1=4000$ when $x=0.60$ m.

The probability density distribution of liquid film thickness (Fig 5) shows a non-Gaussian behaviour that increases with the intermittency factor γ, ie with the Reynolds number Re_L, as observed elsewhere[7-9]. When γ ranges from 0.1 to 0.8, probability density curves have two distinct maxima for film thicknesses b_{pnT} and b_{pT} corresponding to non-turbulent and turbulent phases of liquid film flow. Hence the intermittency phenomenon is well described when the probability density $p(b)$ of the film thickness is written in the form:

$$p(b)=(1-\gamma)p_{nT}(b)+\gamma p_T(b) \tag{1}$$

where $p_{nT}(b)$ is the density probability of b during the non-turbulent phase, and $p_T(b)$ the density probability of b during the turbulent phase.

Mean film thicknesses—ensemble average $\bar{b}$ and conditional averages $\bar{b}_{nT}$ for non-turbulent periods, $\bar{b}_T$ for turbulent periods—are related by the equation:

$$\bar{b}=(1-\gamma)\bar{b}_{nT}+\gamma\bar{b}_T \tag{2}$$

which is verified by measured values. The variations of γ with liquid Reynolds number Re_L shown in Fig 6(a) show a gradual transition to the fully developed turbulence; the representative curve has a point of inflexion for $\gamma=0.5$, marking a flow structure modification.

Evolution of mean period $\bar{T}_B$, lifetime $\bar{\tau}_B$, separation distance $\bar{x}_B$ and length $\bar{l}_B$ of bursts at the film free surface (Figs 6(b) and 6(c)) shows particular behaviour in various ranges:

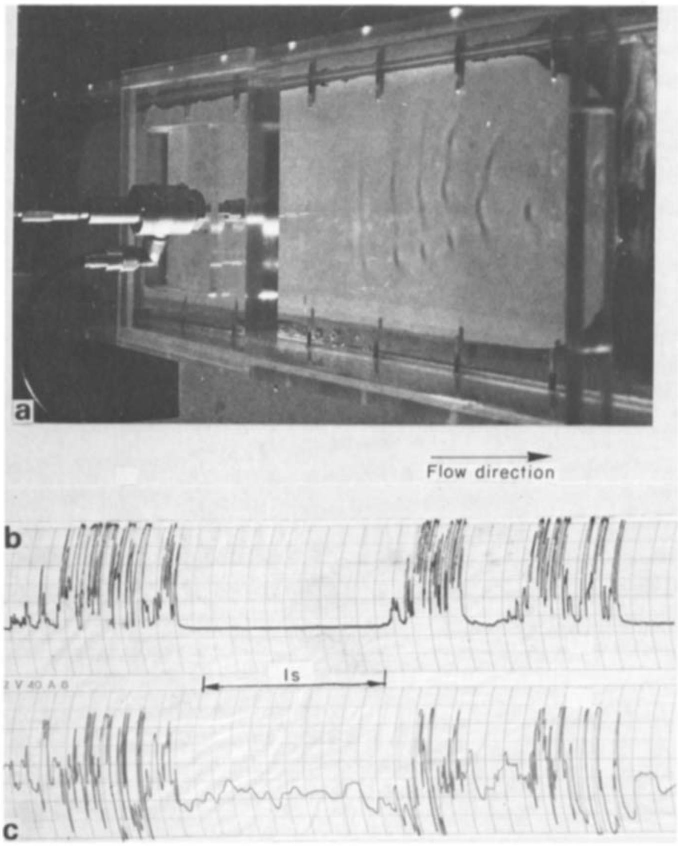
(1) For $0<\gamma<0.3$ ($1800<Re_L<2850$) the mean period $\bar{T}_B$ of burst decreases while their lifetime, $\bar{\tau}_B=\gamma\bar{T}_B$ remains roughly constant. The frequency $1/\bar{T}_B$ of bursting at the free surface increases with increasing eddy generation frequency in the wall region.

(2) For $0.3<\gamma<0.6$ ($2850<Re_L<3200$) the burst period $\bar{T}_B$ is a minimum for γ about 0.5, and the lifetime $\bar{\tau}_B$ of bursts increases, which could be a consequence of vortex stretching and damping of turbulence near the free surface[5].

(3) $0.6<\gamma<1$, ($Re_L>3200$) the mean period $\bar{T}_B$ and lifetime $\bar{\tau}_B$ of bursts increase simultaneously. The mean length $\bar{l}_B$ and separation distance $\bar{x}_B$ of bursts are of the same order. These results could be related to coalescence of bursts, leading to the persistence of the liquid free surface distortions and protuberances. Surface tension forces, described by Weber number We, contribute to vortex stretching, coalescing of eddies and damping of turbulence in the free surface vicinity, stabilizing the film thickness fluctuations (Fig 4).

Notation

$b(t)$	Instantaneous liquid film thickness
$\bar{b}$	Mean film thickness
b'	Root-mean-square value of film thickness fluctuations
c	Capacitance of air-gap
e	Air-gap thickness
f	Oscillatory circuit frequency
Fr	Froude number, $U_d/(g\bar{b})^{1/2}$
g	Gravitational acceleration
$I(x,t)$	Intermittency function
k_L	Transfer coefficient
l	Channel width
l_B	Burst length
N	Number of samples
$p(b)$	Probability density of film thickness b
Q	Liquid discharge per unit channel width
Re_L	Liquid Reynolds number, Q/ν
S	Threshold
t	Time
T	Period
U_d	Liquid discharge velocity, $Q/\bar{b}$
$V(t)$	Instantaneous voltage signal
We	Weber number, $U_d/(\sigma/\rho\bar{b})^{1/2}$
x	Distance from the channel entry
γ	Intermittency factor
ν	Kinematic viscosity of liquid
ρ	Density of liquid
σ	Liquid surface tension
τ	Lifetime of burst



The mean period $\bar{T}$ of turbulent eddies in the viscous sublayer of an incompressible liquid over a plate is a monotically decreasing function of the liquid Reynolds number Re_L. The values of $\bar{T}$ (Fig 6(a)) are of the same order as the results of Kim[10], but distinctly different from those for $\bar{T}_B$ in the film flow.

For energy of film thickness fluctuations, the relation[11]

$$b'^2 = (1-\gamma)b'^2_{nT} + \gamma b'^2_T + \gamma(1-\gamma)(\bar{b}_T - \bar{b}_{nT})^2 \qquad (3)$$

is well verified.

The terms of the left and right sides of Eq (3) are shown in Fig 7(a) versus Re_L, and Fig 7(b) versus γ. The corrective term $\gamma(1-\gamma)$ $(\bar{b}_T - \bar{b}_{nT})^2$, small in comparison with the non-turbulent term $(1-\gamma)b'^2_{nT}$ and turbulent term $\gamma b'^2_T$, is shown in Figs 7(c) and 7(d). It accounts for less than 6% of the total energy b'^2 of liquid film thickness fluctuations (see Appendix).

The structure of film flow is controlled by Reynolds number Re_L, used in the present work to describe the evolution of various characteristics, but two other parameters are: the Froude number $Fr(=U_d/(g\bar{b})^{1/2})$ and Weber number $We(=U_d/(\sigma/\rho\bar{b})^{1/2})$, where U_d is the discharge flow rate of the liquid, σ the surface tension coefficient, ρ the density of the liquid, g the gravitational acceleration, and $\bar{b}$ the mean liquid film thickness. Changes of Froude and Weber numbers, shown in Fig 4, show the gravitational effects stabilize as

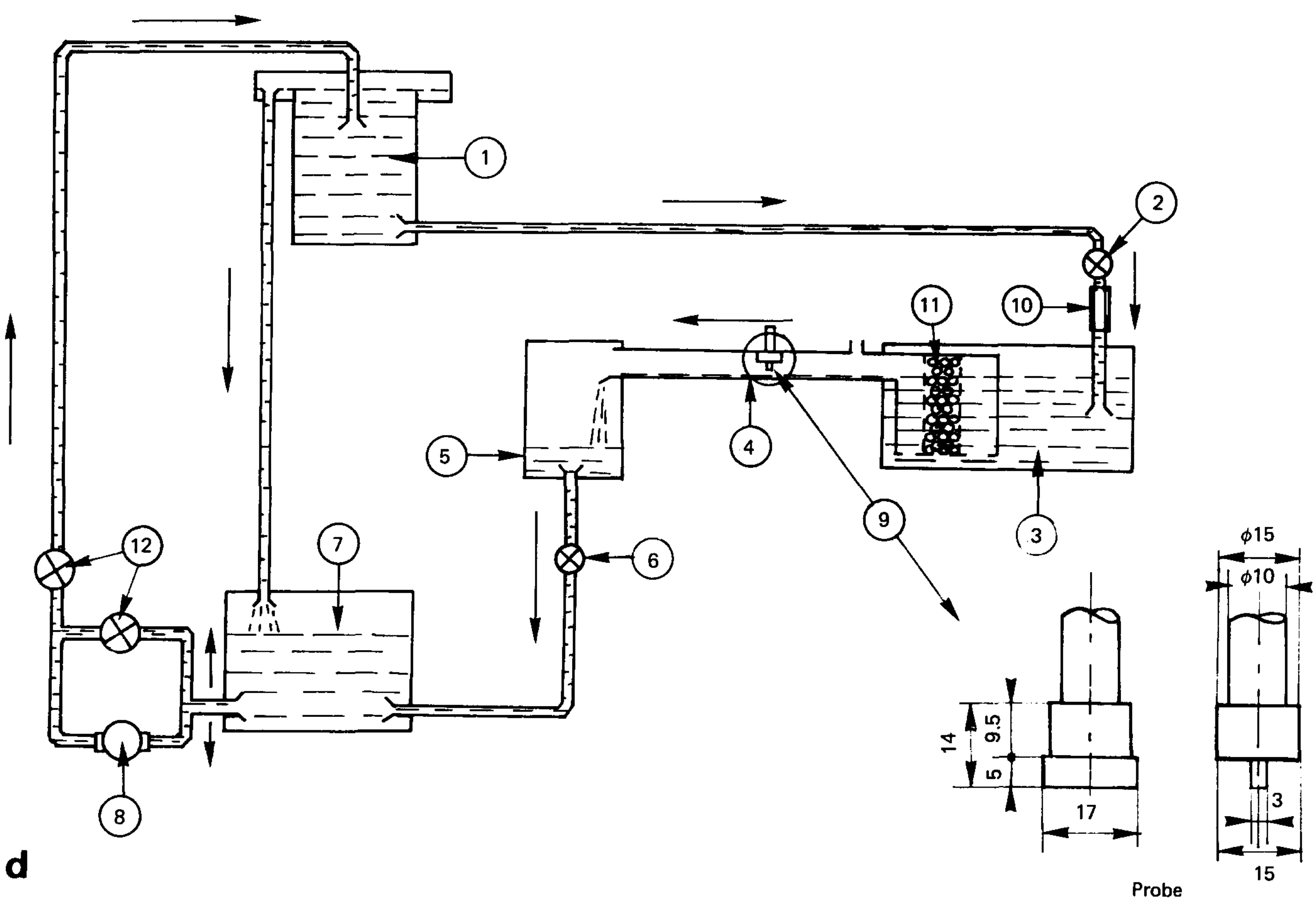


Fig 1 (a) Experimental setup: channel and capacitive probe; (b) full-wave rectified signal after 50–200 Hz filtering; (c) reactance converter output signal V(t) *for Reynolds number* $Re_L = 2854$; *(d) flow system and capacitive probe (dimensions in millimetres: 1, storage tank; 2, 6, 12, valves; 3, 5, 7, reservoirs; 4, channel; 8, pump; 9, capacitive probe; 10 flowmeter; 11, honeycomb*

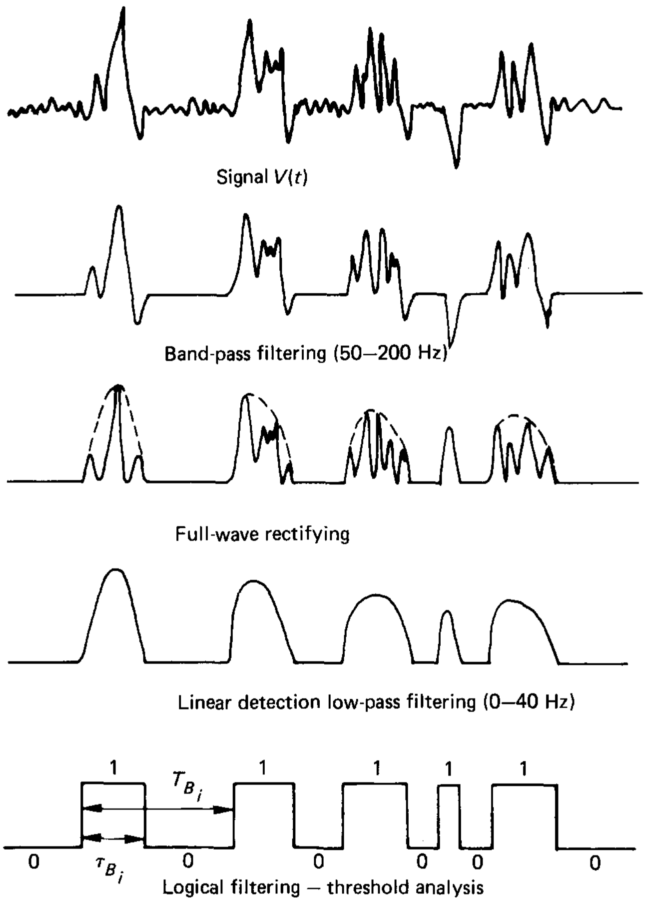


Fig 2 Voltage signal V(t) *processing*

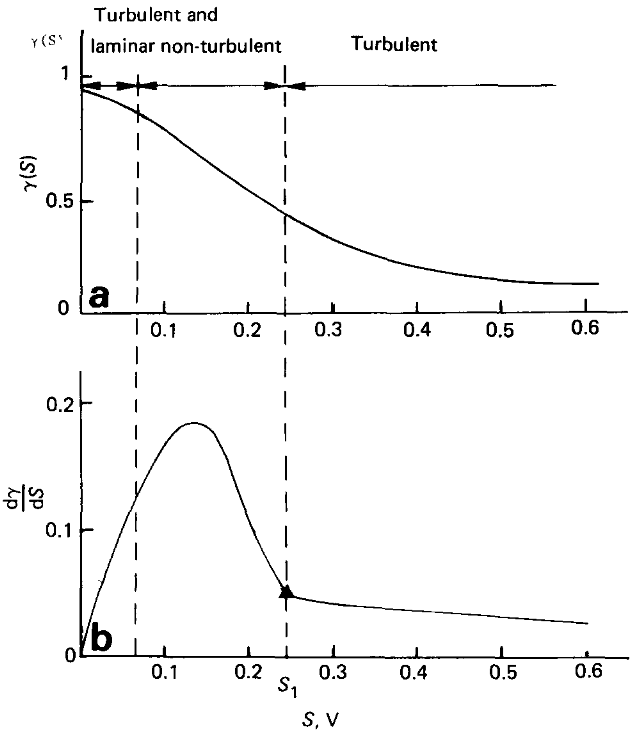


Fig 3 Threshold analysis: (a) probability function of threshold, S; (b) probability density of S

Re_L increases and surface tension effects increase monotically with Re_L.

Conclusions

Liquid film flow is special because of the presence of the free surface, and well-known results of investigations of boundary layers cannot be applied to it. Determination of statistical characteristics in liquid film flow with intermittency could lead to better correlations and agreement between flow parameters and convective transfer results in gas–liquid systems.

Measurements of intermittency factor γ lead to the mean transfer coefficient $\bar{k}_L$ at a liquid film free surface, using the relation

$$\bar{k}_L = (1-\gamma)\bar{k}_{LnT} + \gamma\bar{k}_{LT} \qquad (4)$$

where $\bar{k}_{Lnt}$ is the mean transfer coefficient for non-turbulent phases, as solution of laminar diffusion equations, and $\bar{k}_{LT}$, the mean transfer coefficient for turbulent phases of film flow, as solution of turbulent diffusion equations.

Acknowledgement

We wish to thank M. Billon, CNRS Engineer, for his help in developing measurement techniques, and M. Humeau, CNRS Engineer, for tracings of diagrams.

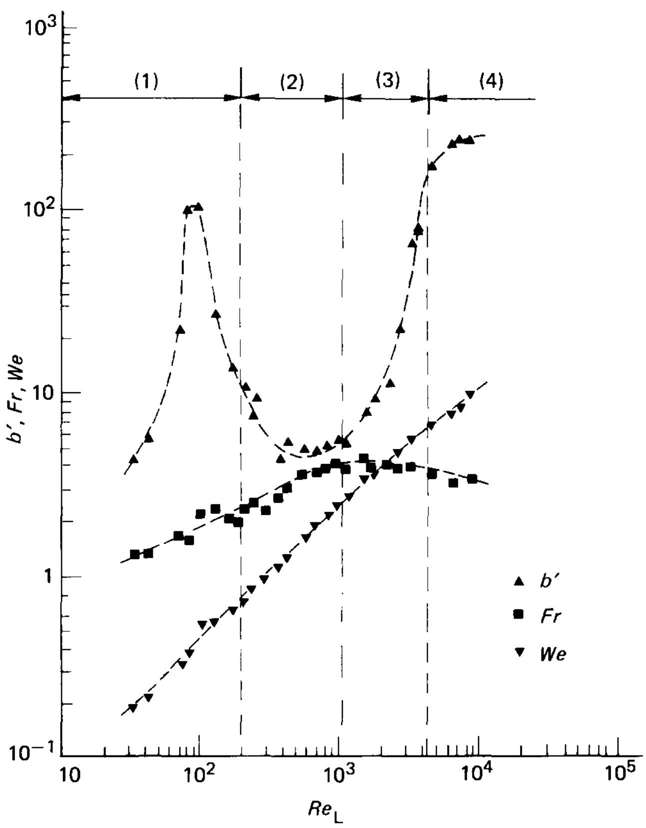
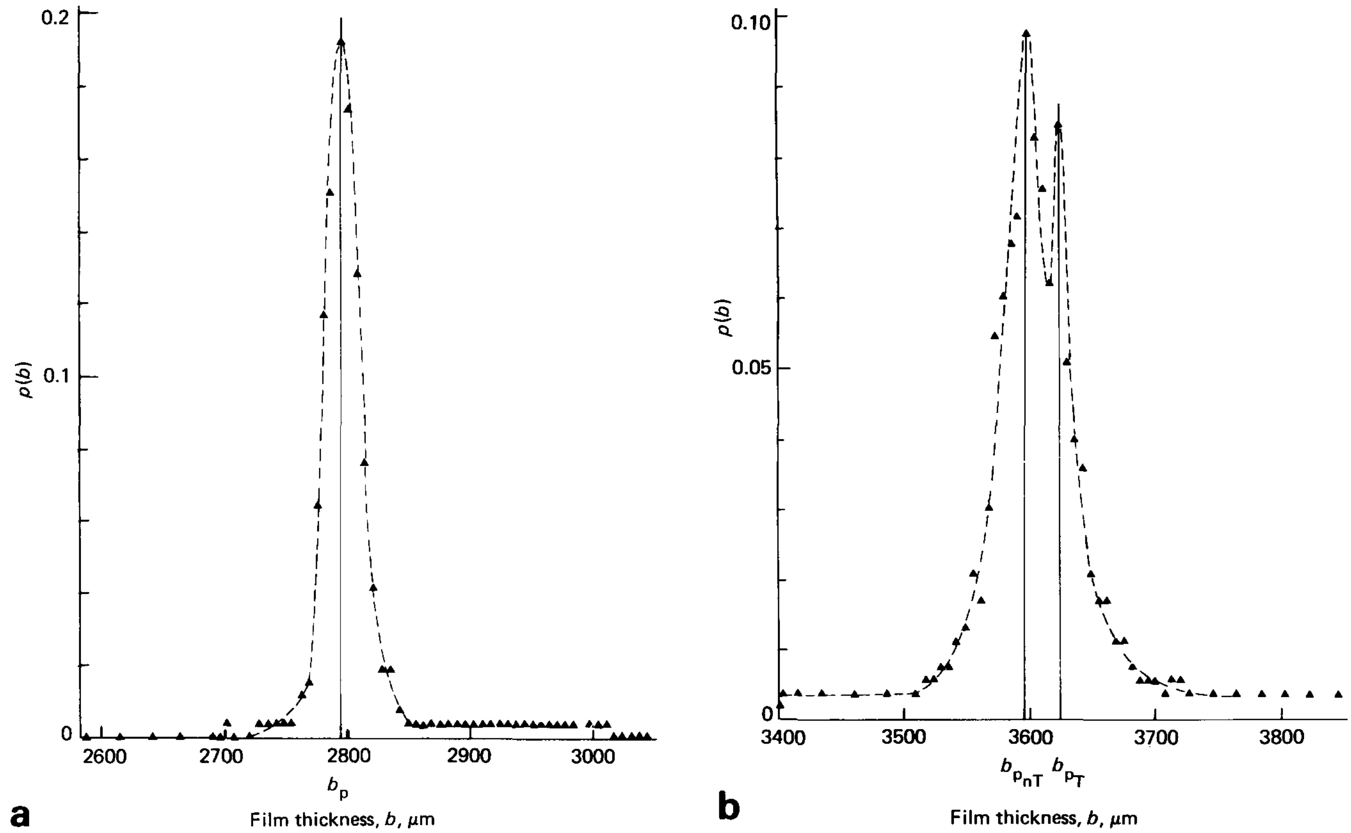
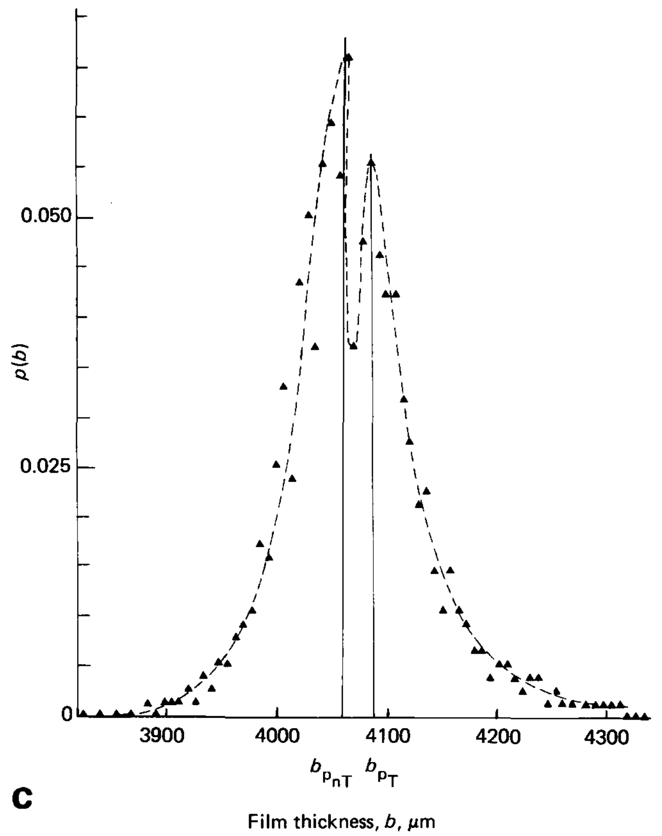


Fig 4 Variation of rms value of film thickness fluctuations, b′, *Froude number* Fr, *and Weber number* We, *with liquid Reynolds number* Re_L





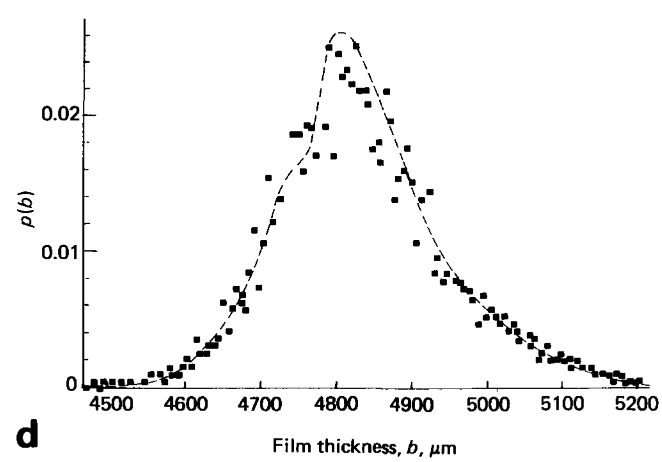


Fig 5 Probability density of film thickness: (*a*) $Re_L=2076$, $\gamma=0.03$, $\bar{b}=2801\ \mu m$, $b'=18\ \mu m$; (*b*) $Re_L=2854$, $\gamma=0.3$, $\bar{b}=3604\ \mu m$, $\bar{b}=45\ \mu m$; (*c*) $Re_L=3288$, $\gamma=0.74$, $b=4074\ \mu m$, $b'=74\ \mu m$; (*d*) $Re_L=3995$, $\gamma=0.94$, $\bar{b}=4844\ \mu m$, $b'=140\ \mu m$

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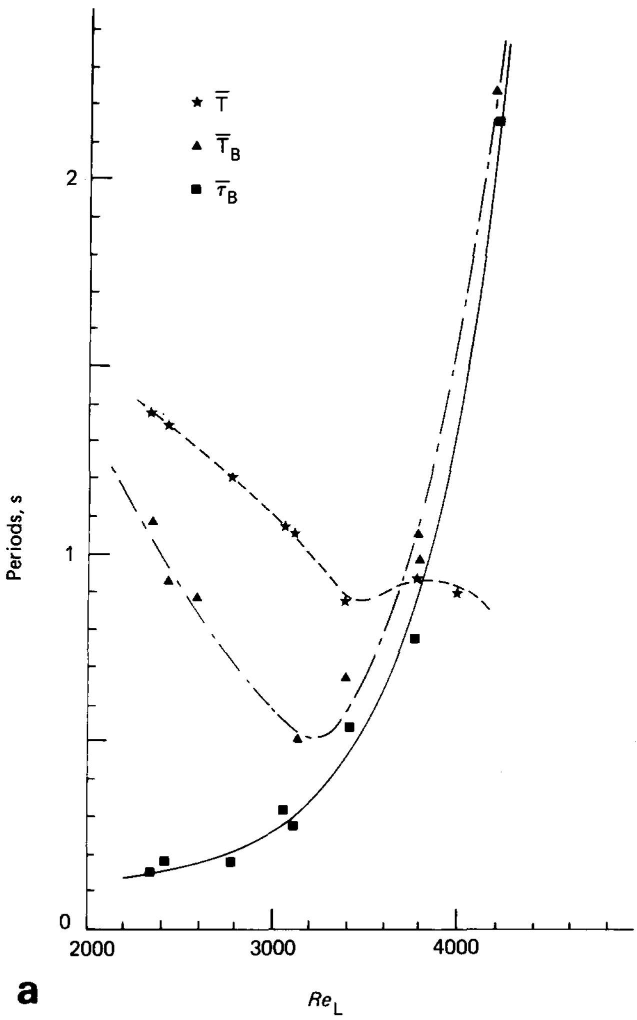

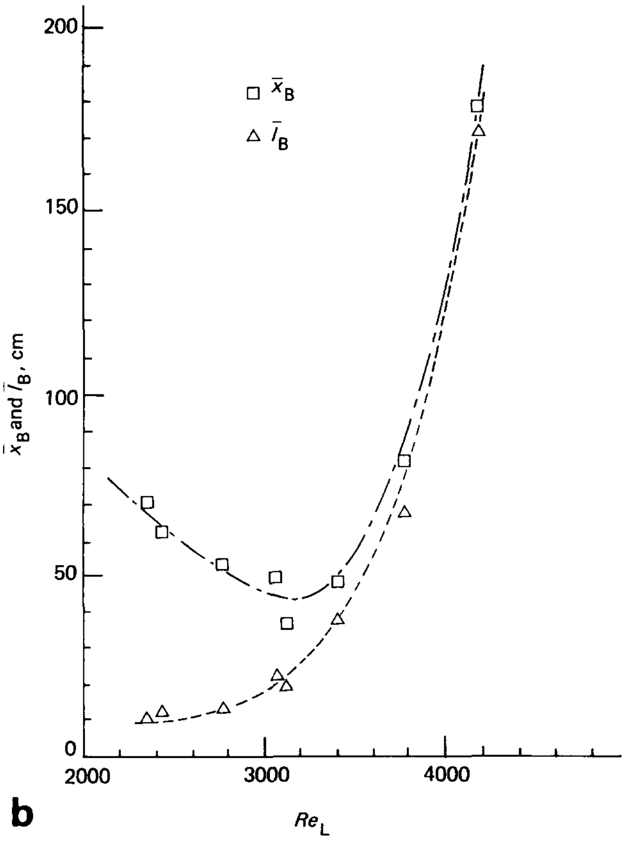

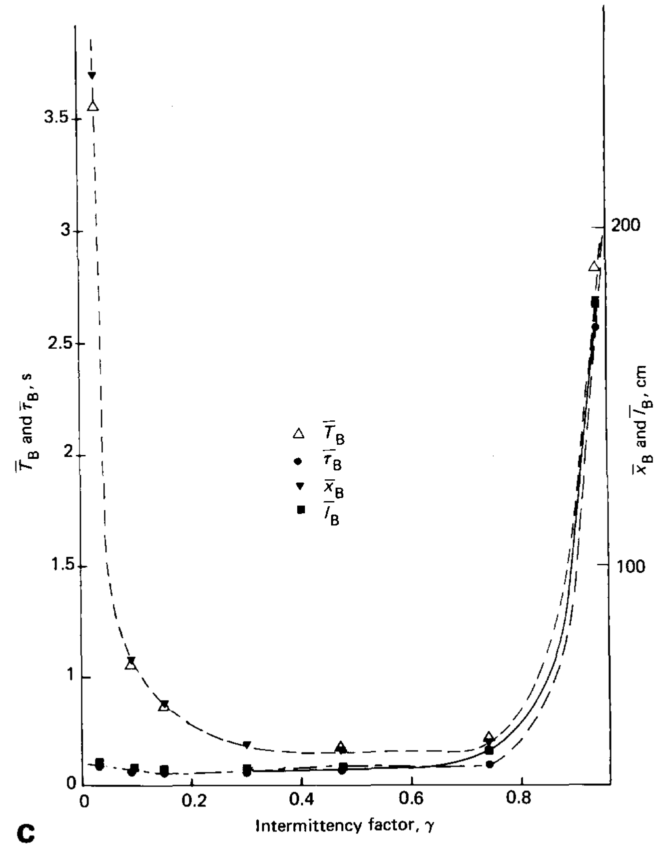

Fig 6 (a) Evolution of mean period $\bar{T}_B$*, and mean lifetime* $\bar{\tau}_B$ *of bursts, and mean period* $\bar{T}$ *of eddies, in a flat plate boundary layer, with liquid Reynolds number* Re_L*; (b) variation of mean separation distance* $\bar{x}_B$ *and mean length* $\bar{l}_B$ *of bursts with liquid Reynolds number* Re_L*; (c) mean period* $\bar{T}_B$*, lifetime* $\bar{\tau}_B$*, separation distance* $\bar{x}_B$ *and length* $\bar{l}_B$ *of bursts versus intermittency factor* γ

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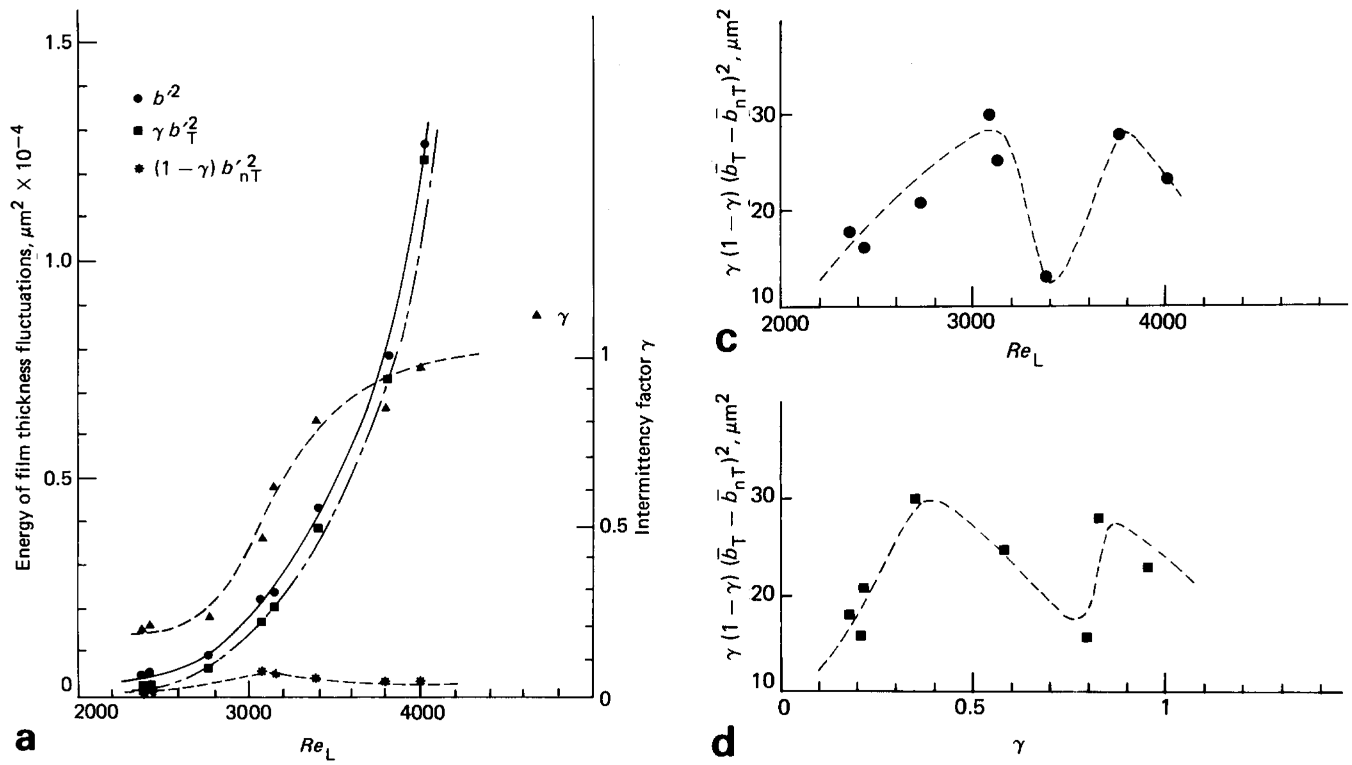
Appendix

Equations (2) and (3)

The probability density of the film thickness $b(t)$ may be written as follows:

$$p(b)=(1-\gamma)p_{nT}(b)+\gamma p_T(b) \qquad \text{(A1)}$$

where γ is the intermittency factor, and $p_{nT}(b)$ and $p_T(b)$ are the probability densities of b in the non-turbulent and



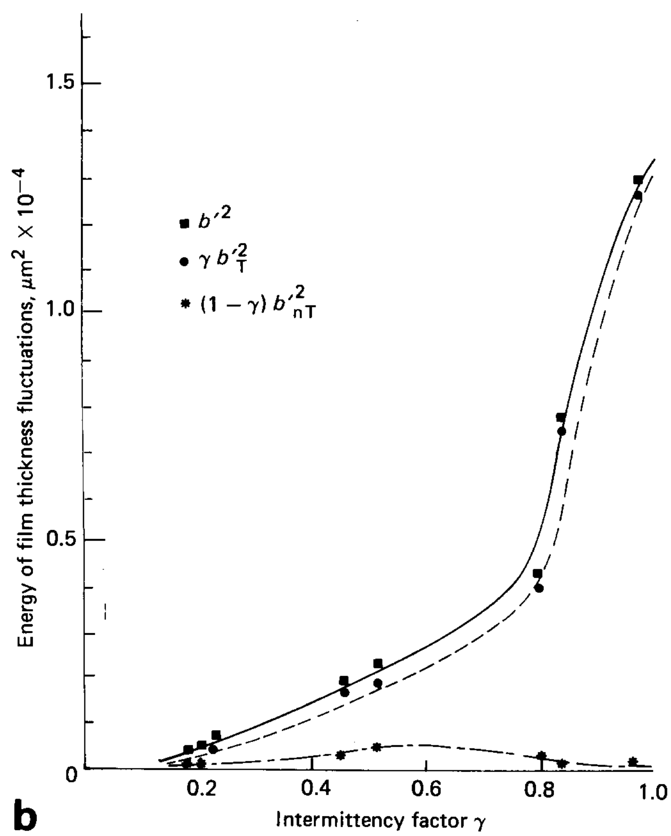


Fig 7 (a) Intermittency factor γ and energy of film thickness fluctuations versus liquid Reynolds number Re_L; (b) energy of film thickness fluctuations versus intermittency factor γ; (c) variation of corrective term $\gamma(1-\gamma)(\bar{b}_T-\bar{b}_{nT})^2$ of film thickness fluctuations with liquid Reynolds number Re_L; (d) variation of corrective term $\gamma(1-\gamma)(\bar{b}_T-\bar{b}_{nT})^2$ of film thickness fluctuations with intermittency factor γ

turbulent regions, respectively. The average film thickness $\bar{b}$ is defined as the first moment of b:

$$\bar{b}=\int_0^\infty bp(b)\,\mathrm{d}b \tag{A2}$$

Eq (A1) and (A2) yield:

$$\bar{b}=\int_0^\infty b[(1-\gamma)p_{\mathrm{nT}}(b)+\gamma p_{\mathrm{T}}(b)]\,\mathrm{d}b$$

$$\bar{b}=(1-\gamma)\int_0^\infty bp_{\mathrm{nT}}(b)\,\mathrm{d}b+\gamma\int_0^\infty p_{\mathrm{T}}(b)\,\mathrm{d}b$$

$$\bar{b}=(1-\gamma)\bar{b}_{\mathrm{nT}}+\gamma\bar{b}_{\mathrm{T}} \tag{A3}$$

Where $\bar{b}_{\mathrm{nT}}$ and $\bar{b}_{\mathrm{T}}$ are the average values in the non-turbulent and turbulent regions:

$$\bar{b}_{\mathrm{nT}}=\int_0^\infty bp_{\mathrm{nT}}(b)\,\mathrm{d}b$$

and

$$\bar{b}_{\mathrm{T}}=\int_0^\infty bp_{\mathrm{T}}(b)\,\mathrm{d}b$$

Eq (2) and (A3) are identical.

For the film thickness fluctuations the rms values b' for the flow, b'_{nT} and b'_{T} for non-turbulent and turbulent zones are defined by

$$\begin{aligned} b'^2&=\mathrm{Var}\,(b)=\overline{b^2}-(\bar{b})^2\\ b'^2_{\mathrm{nT}}&=\overline{b^2_{\mathrm{nT}}}-(\bar{b}_{\mathrm{nT}})^2\\ b'^2_{\mathrm{T}}&=\overline{b^2_{\mathrm{T}}}-(\bar{b}_{\mathrm{T}})^2 \end{aligned} \tag{A4}$$

The second moment of b is

$$\overline{b^2}=\int_0^\infty b^2p(b)\,\mathrm{d}b$$

This last relation and Eq (A1) yield:

$$\overline{b^2}=\int_0^{\infty} b^2[(1-\gamma)p_{nT}(b)+\gamma p_T(b)]\,db$$

$$\overline{b^2}=(1-\gamma)\,\overline{b^2_{nT}}+\gamma\overline{b^2_T} \tag{A5}$$

Hence Eqs (A3), (A4) and (A5) give

$$\overline{b^2}=(1-\gamma)\overline{b^2_{nT}}+\gamma\overline{b^2_T}-[(1-\gamma)\bar{b}_{nT}+\gamma\bar{b}_T]^2$$

$$b'^2=(1-\gamma)\overline{b^2_{nT}}+\gamma\overline{b^2_T}-(1-\gamma)^2(\bar{b}_{nT})^2 - 2\gamma(1-\gamma)\bar{b}_{nT}\bar{b}_T-\gamma^2(\bar{b}_T)^2$$

$$b'^2=(1-\gamma)[\overline{b^2_{nT}}-(\bar{b}_{nT})^2]+\gamma[\overline{b^2_T}-(\bar{b}_T)^2] +(1-\gamma)(\bar{b}_{nT})^2-(1-\gamma)^2(\bar{b}_{nT})^2 +\gamma(\bar{b}_T)^2-\gamma^2(\bar{b}_T)^2-2\gamma(1-\gamma)\bar{b}_{nT}\bar{b}_T$$

$$b'2=(1-\gamma)b'^2_{nT}+\gamma b'^2_T+\gamma(1-\gamma) \times[(\bar{b}_{nT})^2-2\bar{b}_{nT}\bar{b}_T+(\bar{b}_T)^2]$$

$$b'^2=(1-\gamma)b'^2_{nT}+\gamma b'^2_T+\gamma(1-\gamma)(\bar{b}_T-\bar{b}_{nT})^2 \tag{A6}$$

Eqs (3) and (A6) are the same.

Time averages

An intermittency function is defined[11,12] as follows:

$$I(x,t)=\begin{cases}1 \text{ for turbulent flow}\\ 0 \text{ for non-turbulent flow}\end{cases}$$

The time average of $I(x,t)$ yields the intermittency factor

$$\gamma=\bar{I}=\lim_{(T_0\to\infty)}\frac{1}{T_0}\int_{t_0}^{t_0+T_0} I(x,t)\,dt \tag{A7}$$

The time average of any function $q(t)$, denoted by $\bar{q}$ is given by

$$\bar{q}=\lim_{(T_0\to\infty)}\frac{1}{T_0}\int_{t_0}^{t_0+t_0} q(t)\,dt \tag{A8}$$

The zone averages in the non-turbulent and turbulent regions q_{nT} and q_T are defined as

$$q_{nT}=\lim_{(T_0\to\infty)}\frac{1}{(1-\gamma)T_0}\int_{t_0}^{t_0+T_0}[1-I(x,t)]q(t)\,dt \tag{A9}$$

and

$$q_T=\lim_{(T_0\to\infty)}\frac{1}{T_0}\int_{t_0}^{t_0+T_0} I(x,t)q(t)\,dt \tag{A10}$$

where t_0 is the time when averaging begins.

For a time sequence $t_1, t_2, \ldots, t_N$ in which every point t_i $(i=1,2,\ldots,N)$ denotes the temporal reference of the event of interest, the ensemble average of the quantity $q(t)$ related to the event is given by

$$\langle q(x,\theta_T)\rangle=\frac{1}{N}\sum_{i=1}^{N} q(x,t_i+\theta_T) \tag{A11}$$

where x denotes the position of the sampling probe and θ_T is the time when the fluid either enters or leaves the turbulent phase.

The substitution of sampled averages, determined by the $V(t)$ signal processing (Fig 2) in Eqs (2) and (3) shows that those relations are well verified. We make the assumption that if mass transfer takes place at the free surface of a liquid film in flow with intermittency, the average mass transfer coefficient $\bar{k}_L$ could verify Eq (4) in a similar way to Eq (2) for the average film thickness.

Threshold analysis

The technique of threshold analysis has been described by Chen[13] and Blackwelder[14]. It consists of varying the threshold level over a large range of the film thickness b corresponding to a large range of signal $V(t)$. Fig 3 shows the interface of turbulent/non-turbulent phases. The threshold S_1 yields the exact value of γ, checking with that determined by logical filtering as the ensemble average of $\gamma_i=\tau_{B_i}/T_{B_i}$, that is:

$$\gamma=\frac{1}{N}\sum_{i=1}^{N}\gamma_i \tag{A12}$$

The probe used for measurement is connected to the DISA Capacitive Measurement System[15] consisting of proximity transducer (type 51D11), an oscillator and a reactance converter (type 51E01). This sytem has a high sensitivity.

The air-gap thickness between the probe and the free liquid film surface ranges from 0.4 to 0.8 mm when the film thickness changes from 3 to 5 mm.

Books received

Basic Programs for steam plant engineers, *V. Ganapathy*, \$39.75 (USA and Canada only) \$47.50 (all other countries), pp 168, Marcel Dekker

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